

Algorithm I (*Treap Insertion*). Given a set of nodes which form a treap T , and a key to insert K , this algorithm will insert the node into the treap while maintaining its heap properties. Each node is assumed to contain KEY , $PRIO$, $LLINK$, $RLINK$, and $PARENT$ fields. For any given node N , $KEY(N)$ gives the key field of N , $PRIO(N)$ gives the priority field of N , $LLINK(N)$ and $RLINK(N)$ are pointers to N 's left and right subtrees, respectively, and $PARENT(N)$ is a pointer to the node of which N is a subtree. Any or all of these three link fields may be Λ , which for $LLINK(N)$ and $RLINK(N)$ indicates that N has no left or right subtree, respectively, and for $PARENT(N)$ indicates that N is the root of the treap. The treap has a field $ROOT$ which is a pointer to the root node of the treap.

You can find an implementation of this algorithm, as well as many others, in **libdict**, which is available on the web at <http://www.crazycoder.org/libdict.html>.

- I1.** [Initialize.] Set $N \leftarrow ROOT(T)$, $P \leftarrow \Lambda$.
- I2.** [Find insertion point.] If $N = \Lambda$, go to step I3. If $K = KEY(N)$, the key is already in the treap and the algorithm terminates with an error. Set $P \leftarrow N$; if $K < KEY(N)$, then set $N \leftarrow LLINK(N)$, otherwise set $N \leftarrow RLINK(N)$. Repeat this step.
- I3.** [Insert.] Set $N \leftarrow AVAIL$. If $N = \Lambda$, the algorithm terminates with an out of memory error. Set $KEY(N) \leftarrow K$, $LLINK(N) \leftarrow RLINK(N) \leftarrow \Lambda$, and $PARENT(N) \leftarrow P$. Set $PRIO(N)$ equal to a random integer. If $P = \Lambda$, set $ROOT(T) \leftarrow N$, and go to step I5. If $K < KEY(P)$, set $LLINK(P) \leftarrow N$; otherwise, set $RLINK(P) \leftarrow N$.
- I4.** [Sift up.] If $P = \Lambda$ or $PRIO(P) \leq PRIO(N)$, go to step I5. If $LLINK(P) = N$, rotate P right; otherwise, rotate P left. Then set $P \leftarrow PARENT(N)$, and repeat this step.
- I5.** [All done.] The algorithm terminates successfully.

Rotations

Algorithm R (*Right Rotation*). Given a treap T and a node in the treap N , this routine will rotate N right.

- R1.** [Do the rotation.] Set $L \leftarrow LLINK(N)$ and $LLINK(N) \leftarrow RLINK(L)$. If $RLINK(L) \neq \Lambda$, then set $PARENT(RLINK(L)) \leftarrow N$. Set $P \leftarrow PARENT(N)$, $PARENT(L) \leftarrow P$. If $P = \Lambda$, then set $ROOT(T) \leftarrow L$; if $P \neq \Lambda$ and $LLINK(P) = N$, set $LLINK(P) \leftarrow L$, otherwise set $RLINK(P) \leftarrow L$. Finally, set $RLINK(L) \leftarrow N$, and $PARENT(N) \leftarrow L$.

The code for a left rotation is symmetric. At the risk of being repetitive, it appears below.

Algorithm L (*Left Rotation*). Given a treap T and a node in the treap N , this routine will rotate N left.

- L1.** [Do the rotation.] Set $R \leftarrow RLINK(N)$ and $RLINK(N) \leftarrow LLINK(R)$. If $LLINK(R) \neq \Lambda$, then set $PARENT(LLINK(R)) \leftarrow N$. Set $P \leftarrow PARENT(N)$, $PARENT(R) \leftarrow P$. If $P = \Lambda$, then set $ROOT(T) \leftarrow R$; if $P \neq \Lambda$ and $LLINK(P) = N$, set $LLINK(P) \leftarrow R$, otherwise set $RLINK(P) \leftarrow R$. Finally, set $LLINK(R) \leftarrow N$, and $PARENT(N) \leftarrow R$.