Radiroot

Roots Of A Polynomial By Radicals

A GAP4 Package

Version 1.0

by

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1 Introduction

The main functionality of this package is to solve a rational polynomial by radicals and display the solution. That is possible iff the Galois group of the polynomial – a permutation group on its roots – is solvable. This fact has first been discovered by Évariste Galois (1811 – 1832), on whose ideas this implementation is based. The implemented algorithm is discribed in [Dis05].

The package creates a LaTex file for the radical expression. Therefore you need a Latex compiler and the dvi viewer xdvi, to use the main functionality.

In addition to the readout you get several results in GAP. Some of them can be computed on their own. This are the splitting field of a rational polynomial and its Galois group as a permutation on the roots.

This package uses the interface to KANT [DFK+97] in the package Alnuth to factorize polynomials over algebraic numberfields. This functionality must be available to use the functions in Radiroot.

2 Functionality of the package

2.1 Methods for rational polynomials

1 ▶ `IsSolvable(` *f* `)`
▶ `IsSolvablePolynomial(` *f* `)`

returns `true` if the rational polynomial f has a solvable Galois group and `false` otherwise. It signals an error if there exists an irreducible factor with degree greater than 15.

For a rational polynomial f

2 ▶ `SplittingField(` *f* `)`

returns the smallest field, constructed with `FieldByPolynomial`, that contains all roots of f.

3 ▶ `GaloisGroupOnRoots(` *f* `)`

calculates the Galois group G of the rationals polynomial f as a permutation group with respect to the ordering of the roots of f given as matrices in *G!.roots*.

2.2 Solving a polynomial by radicals

1 ▶ `RootsOfPolynomialAsRadicals(` *f* `)`

computes a solution by radicals for the irreducible, rational polynomial f up to degree 15 if this is possible, e. g. if the Galoisgroup of f is solvable. The result is displayed in form of a dvi-file. Additionally a record is returned which contains the roots of f as a list `roots` of matrices, the Galois group on the roots as component `galgrp` and the splitting field of f in two forms; on the one hand the matrix field `K` generated by the roots and on the other hand an algebraic number field `H` created by the defining polynomial of the matrix field. The record also includes a list `cyclics` of matrices which define the splitting field by gradual, cyclic extensions.

The computation may last very long and doesn't finish for every example if the degree of f is greater than 7.

2 ▶ `RootsOfPolynomialAsRadicalsNC(` *f*`,` *display* `)`

does essentially the same as `RootsOfPolynomialAsRadicals` except that you can choose if you want to create a dvi-file and display it by setting the boolean *display*. The function performs no test whether the polynomial f is irreducible. It also doesn't check at the begining if f is solvable, but can therefore be used for polynomials with arbitrary degree.

3 Installation

3.1 Getting and Installing this Package

This package is available at

```
http://www.icm.tu-bs.de/ag_algebra/software/distler/radiroot
```

in form of a gzipped tar-archive. For the installation instructions see Chapter 74.1 in the GAP Reference Manual. Normally you will unpack the archive in the 'pkg' directory of your GAP-Version by typing:

```
bash> tar xfz radiroot.tar.gz        # for the gzipped tar-archive
```

3.2 Loading the Package

To use the Radiroot Package you have to request it explicitly. This is done by calling

```
gap> LoadPackage("radiroot");
-------------------------------------------------------------------------------
Loading  RadiRoot 1.0 (Roots of a Polynomial as Radicals)
by Andreas Distler (a.distler@tu-bs.de).
-------------------------------------------------------------------------------
true
```

The LoadPackage command is described in Section 74.2.1 in the GAP Reference Manual.

If you want to load the Radiroot package by default, you can put the LoadPackage command into your .gaprc file (see Section 3.4 in the GAP Reference Manual).

Bibliography

[DFK+97] M. Daberkow, C. Fieker, J. Klüners, M. Pohst, K.Roegner, and K. Wildanger. Kant V4. *J. Symb. Comput.*, 24:267 – 283, 1997.

[Dis05] Andreas Distler. Ein Algorithmus zum Lösen einer Polynomgleichung durch Radikale. Diplomarbeit, TU Braunschweig, 2005.

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This index covers only this manual. A page number in *italics* refers to a whole section which is devoted to the indexed subject. Keywords are sorted with case and spaces ignored, e.g., "`PermutationCharacter`" comes before "permutation group".